ANALYSIS OF HIGHER-ORDER N-TONE SIGMA-DELTA MODULATORS FOR ULTRA WIDEBAND COMMUNICATIONS

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ABSTRACT

We consider the properties of a class of non-oversampling N-tone sigma-delta modulators which have applications in the design of UWB-OFDM communications systems. The spectrum gaps that exist in such systems are well-matched to the noise shaping properties of these modulators and their non-oversampling nature makes them practical for use with these ultra wideband signals. Performance results for first-order, second-order and L^{th}-order modulators are presented, and a general expression for the excess resolution that can be gained in such systems is obtained.

1. INTRODUCTION

Ultra Wideband Orthogonal Frequency Division Multiplexing (UWB-OFDM) is a fast frequency hopping multi-carrier system that uses frequency coded pulse trains to achieve high multipath resolution. Traditional UWB communication uses either pulse amplitude or pulse position modulation without any carrier [1-3]. In 2002, the FCC allocated the 3.1-10.6 GHz band for UWB systems, and UWB-OFDM has been proposed to efficiently utilize this spectrum [4-6].

A UWB-OFDM signal consists of N orthogonal sub-carriers modulated with N BPSK, QPSK or M-ary QAM symbols in parallel. High-speed, high resolution D/A and A/D converters are required to generate and recover these signals. Techniques based on traditional sigma-delta data converters could be considered, but that would require oversampling an ultrawide bandwidth signal [7]. An attractive alternative is to use an N-tone sigma-delta modulator because it operates without the need for oversampling [8]. In particular, we consider a system in which only a subset of the orthogonal sub-carriers are used to send information. This results in a spectrum containing gaps, which matches well with the noise-shaping properties of the N-tone sigma-delta modulator.

In this paper, we analyze the performance of these systems and obtain results for the excess resolution gains that can be achieved. Since these are non-oversampled structures, it is interesting to relate this metric to that of a traditional oversampled sigma-delta modulator, and we include this as part of our analysis. We consider first-order and second-order systems explicitly and then generalize the results to an L^{th}-order system. We also present simulation results for the magnitude response of these structures.

2. SIGMA-DELTA MODULATORS

The traditional sigma-delta conversion technique uses oversampling and noise shaping to reduce the quantization noise in the band of interest. Oversampling reduces the quantization noise, which is assumed to be uniformly distributed between $-\Delta V/2$ and $\Delta V/2$ (where ΔV is the quantization step size) in the lowpass signal band of interest ($-F_M \le f \le F_M$) [9]. By increasing the sampling frequency, F_T, the quantization noise power can be spread over F_T, which is much larger than the signal band. It is useful to compare the number of bits, B, needed for a Nyquist rate converter for which its in-band quantization noise is equal to that of a b_{os}-bits converter operating at a higher rate [9]:

$$B - b_{os} = \frac{1}{2}\log_2 M \qquad (4)$$

where $M = F_T / 2F_M$ denotes the oversampling ratio. Equation (4) gives the excess resolution gained by oversampling at M times the Nyquist rate. This implies that the increase in resolution is 0.5 bits for each doubling of the oversampling ratio M.

Figure 1 shows the structure of a traditional sigma-delta modulator. Since the feedback loop contains a nonlinear element, its exact analysis is difficult so a discrete linear model is usually used instead [9]. Figure 2 shows the corresponding linear model where e[n] represents the uniformly distributed quantization noise. The output of the sigma-delta modulator, Y(Z), is related to the input X(Z) and the noise E(Z) as follows:

$$Y(Z) = X(Z) + (1 - Z^{-1})E(Z) \qquad (5)$$

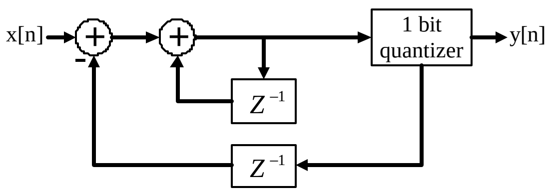

Figure 1: Sigma-delta modulator.

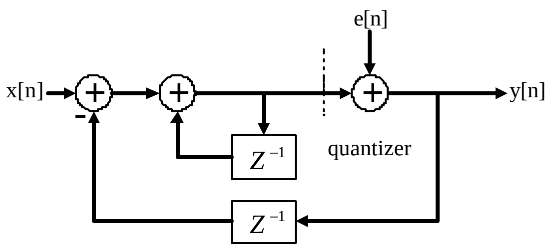

Figure 2: Discrete-time model of the sigma-delta modulator.

The noise shaping filter, $G(Z) = 1 - Z^{-1}$, attenuates the quantization noise in the signal band and amplifies it outside of this band. It can be viewed as pushing quantization noise power away from the signal band to higher frequencies. The same kind of analysis can be done in order to obtain the excess resolution for this case:

$$B - b_{\Sigma\Delta} = \frac{1}{2}\log_2[Mp] - \frac{1}{2}\log_2[2p - 2M\sin(\frac{p}{M})] \qquad (6)$$

If the signal band is much smaller than sampling frequency ($F_M << F_T$), equation (6) can be simplified as:

$$B - b_{\Sigma\Delta} \cong \log_2 \frac{\sqrt{3}}{p} + \frac{3}{2}\log_2 M \qquad (7)$$

The increase in the resolution is about 1.5 bits per doubling of the oversampling ratio M. A sigma-delta modulator is usually used when the input is a lowpass signal and it attains its highest resolution for relatively low signal bandwidth [10].

A UWB-OFDM signal can be viewed as collection of narrowband signals centered at the sub-channel frequencies [8]. Therefore, the noise shaping filter of a traditional sigma-delta modulator must be modified in order to remove quantization noise in these sub-channel frequencies. The N-tone sigma-delta introduced in Ref. [8] can achieve this desired noise shaping. Figure 3 shows the structure of an N-tone sigma-delta modulator and the corresponding discrete-time linear model of the system can be expressed as follows:

$$Y(Z) = X(Z) + (1 - Z^{-N})E(Z) \qquad (8)$$

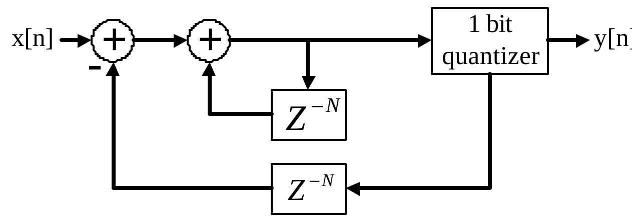

Figure 3: N-tone sigma-delta modulator.

The noise shaping filter, $G_N(Z) = 1 - Z^{-N}$, is a comb filter with notches at frequencies $2p\, l/N$; $l = 0, 1, \ldots, N-1$ and thus it can be used for a signal that has narrowband frequency components at these locations. The frequency response of $G_N(Z)$ is shown in Figure 4. The in-band noise can be obtained assuming that K of these N notches have narrowband frequency signal components and each has a bandwidth of $2F_{SC}$.

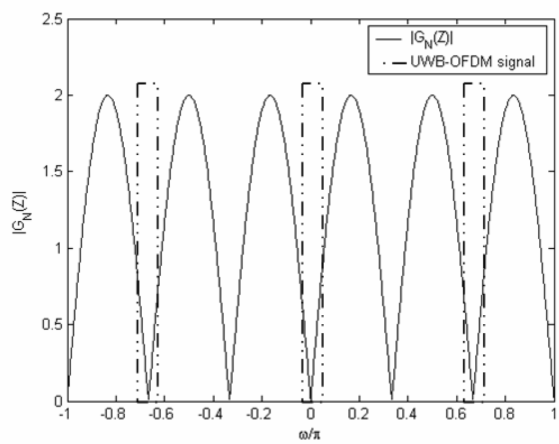

Figure 4: Frequency response of the noise shaping filter of an N-tone sigma-delta modulator with an OFDM signal.

The excess bit resolution gained from using an N-tone sigma-delta for UWB-OFDM signals is calculated to be:

$$B - b_{N-\Sigma\Delta} = \frac{1}{2}\log_2[NM'p] - \frac{1}{2}\log_2[2KNp - 2KM'\sin(\frac{pN}{M'})] \qquad (9)$$

where $M' = F_T / 2F_{SC}$. If $F_{SC} << F_T / N$, this equation can be approximated as follows:

$$B - b_{N-\Sigma\Delta} \cong \log_2 \frac{\sqrt{3}}{p} + \frac{3}{2}\log_2[\frac{M'}{\sqrt[3]{KN^2}}] \qquad (10)$$

Comparing equations (7) and (10), we can see that a similar resolution gain is achieved in our non-oversampled N-tone sigma-delta UWB-OFDM system as that of a traditional oversampling sigma-delta modulator operating in a lowpass system with $F_T = 2F_M M$ provided that $M'/\sqrt[3]{KN^2} \approx M$.

To further attenuate the in-band noise for OFDM systems, a higher-order N-tone sigma-delta modulator can be used. Figure 5 shows the case of a second-order N-tone sigma-delta structure. Its input-output-noise relationship is given as follows:

$$Y(Z) = X(Z) + (1 - Z^{-N})^2 E(Z) \tag{11}$$

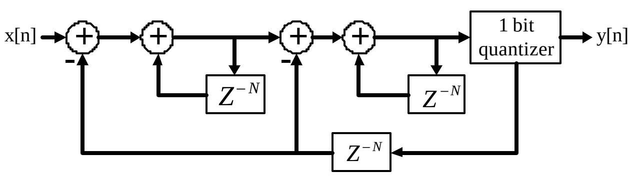

Figure 5: Second-order N-tone sigma-delta modulator.

The noise shaping filter, $G_{N2}(Z) = (1 - Z^{-N})^2$, attenuates in-band noise stronger than the filter $G_N(Z)$ used in the first-order case. The frequency response of $G_{N2}(Z)$ has deeper notches than $G_N(Z)$ at same frequency locations $2p\, l/N\,;\, l = 0, 1, \ldots, N-1$. The excess bit resolution gained from using a second-order N-tone sigma-delta for UWB-OFDM signal is found to be:

$$B - b_{N2-\Sigma\Delta} = \frac{1}{2}\log_2[NM'p] - \frac{1}{2}\log_2[6KNp - 2KM'\left(\sin(\frac{2pN}{M'}) - 8\sin(\frac{pN}{M'})\right)] \tag{12}$$

This can be approximated when $F_{SC} << F_T / N$ as follows:

$$B - b_{N2-\Sigma\Delta} \cong \log_2\left(\frac{\sqrt{5}}{p^2}\right) + \frac{5}{2}\log_2[\frac{M'}{\sqrt[5]{KN^4}}] \tag{13}$$

Comparing equations (10) and (13), we see that the increase in resolution is about 2.5 bits per doubling of $M'/\sqrt[5]{KN^4}$ for the second-order design instead of 1.5-bits per doubling of $M'/\sqrt[3]{KN^2}$ for the first-order case.

A general Lth-order N-tone sigma-delta modulator can be constructed by cascading L similar sections, as shown in Figure 6. Its input-output-noise relationship is as follows:

$$Y(Z) = X(Z) + (1 - Z^{-N})^L E(Z) \tag{14}$$

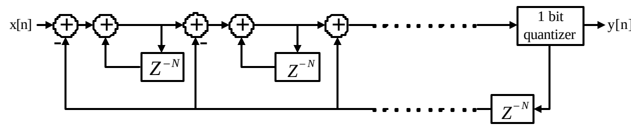

Figure 6: General Lth-order N-tone sigma-delta modulator.

The Lth-order N-tone sigma-delta modulator uses a noise shaping filter $G_{NL}(Z) = (1 - Z^{-N})^L$. The excess bit resolution gained from this structure can be approximated when $F_{SC} << F_T / N$ as follows:

$$B - b_{NL-\Sigma\Delta} \cong \log_2\left(\frac{\sqrt{2L+1}}{p^L}\right) + \frac{2L+1}{2}\log_2[\frac{M'}{\sqrt[2L+1]{KN^{2L}}}]$$

(15)

From the above equation, we can see that the increase in the excess resolution is about $(2L+1)/2$ bits per doubling of $M'/\sqrt[2L+1]{KN^{2L}}$. The increased performance that can be achieved using higher-order N-tone sigma-delta modulators must be traded off against the increase in the system complexity as well as a longer critical path delay. Furthermore, the use of multiple feedback loops may result in an unstable system so proper care must be taken to ensure that stability is achieved [9].

3. SIMULATION RESULTS

A traditional sigma-delta modulator was simulated with a lowpass sinusoidal input. The magnitude response of the output of the modulator is shown in Figure 7. The quantization noise is pushed away from the lowpass signal band to higher frequencies and the input signal can be recovered by passing the output through a low pass filter to remove the high frequency quantization noise.

The magnitude response of a first-order N-tone sigma-delta modulator is shown in Figure 8. Note that the quantization noise spectrum has notches at frequencies $2p\, l/N\,;\, l = 0, 1, \ldots, N-1$ as described earlier. The existence of these notches can be exploited to transmit the UWB-OFDM sub-channel signals that are placed at those locations. At the receiver, the signals can be recovered by passing the output of the modulator through a comb notch filter to remove the quantization noise.

modulators comes at the cost of increased system complexity and a longer critical path delay.

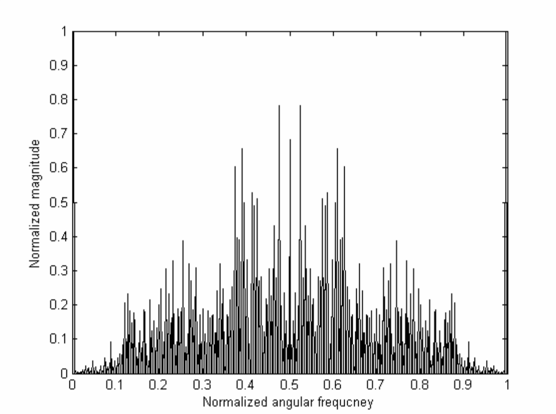

Figure 7: Magnitude response of a traditional sigma-delta modulator

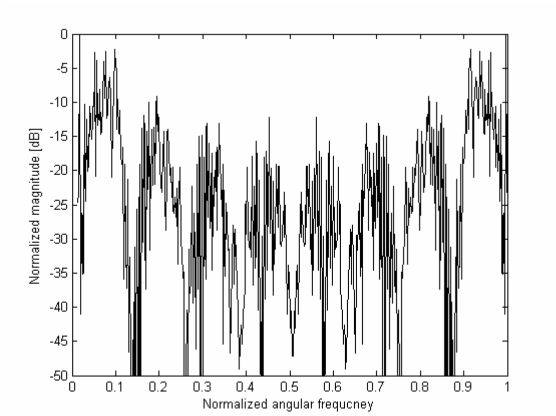

Figure 8: Magnitude response of a first-order N-tone sigma-delta modulator

A better performance is obtained by using a second-order modulator with deeper notches as shown in Figure 9. We observe that the quantization noise is pushed further away from the notches, thus improving the signal to quantization noise ratio. As can be seen, the notches for the second-order N-tone sigma-delta modulator are better defined then those of the first-order design.

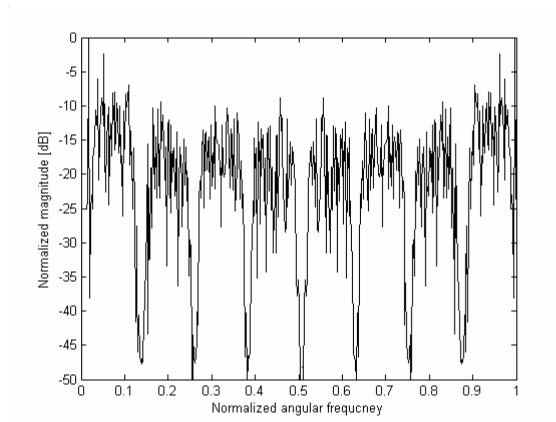

Figure 9: Magnitude response of a second-order N-tone sigma-delta modulator

4. CONCLUSIONS

Due to the high data rates involved and the characteristics of the spectrum of UWB-OFDM communications systems, non-oversampled N-tone sigma-delta modulators are found to be suitable for use in these applications. We have analyzed the performance of first-order, second-order and general-order N-tone modulators in terms of their excess bit resolution gains. The better performance that is obtained using higher-order N-tone sigma-delta modulators comes at the cost of increased system complexity and a longer critical path delay.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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